Spatial prisoner's dilemma game with volunteering in Newman-Watts small-world networks

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A modified spatial prisoner's dilemma game with voluntary participation in Newman-Watts small-world networks is studied. Some reasonable ingredients are introduced to the game evolutionary dynamics: each agent in the network is a pure strategist and can only take one of three strategies (*cooperator, defector*, and *loner*); its strategical transformation is associated with both the number of strategical states and the magnitude of average profits, which are adopted and acquired by its coplayers in the previous round of play; a stochastic strategy mutation is applied when it gets into the trouble of *local commons* that the agent and its neighbors are in the same state and get the same average payoffs. In the case of very low temptation to defect, it is found that agents are willing to participate in the game in typical small-world region and intensive collective oscillations arise in more random region.

There has been a long history of studying complex behaviors qualitatively of biological, ecological, social, and economic systems using special game models. After the prisoner's dilemma game (PDG) was first applied by Neumann and Morgenstern [1] to study economic behavior, great developments have been made by a lot of subsequent studies. Recently, more and more attention has been focused on the applications of the PDG in the fields of biology [2], economy [3], ecology [4], and other domains [5]. Game theory and evolutionary theory provide a powerful metaphor for simulating the interactions of individuals in these systems [6].

Most realistic systems can be regarded as composed of a large number of individuals with simple local interactions. For example, human beings are limited in territory and interact more frequently with their neighbors than those far away. Therefore, the spatial structure may greatly affect their activities. Since Axelrod [7] suggested ideas of the PDG on a lattice, spatial prisoner's dilemma games (SPDG) have been extensively explored in various kinds of network models in the past few years, including regular lattices [8–10], random regular graphs [11], random networks with fixed mean degree distribution [12], small-world networks [13–15], and real-world acquaintance networks [16], etc. In the general SPDG, each agent can take one of two strategies (or states): *cooperator* (*C*) and *defector* (*D*). There are four possible combinations: (C,C), (C,D), (D,C), and (D,D), which get payoffs (r,r), (s,t), (t,s), and (p,p), respectively. The parameters satisfy the conditions $t>r>p>s$ and $2r>t+s$, so that leads to a so-called dilemma situation where mutual trust and cooperation are beneficial in a long perspective but egoism and guile can produce big short-term profit. Agents update their states by imitating the strategy of the wealthiest among their neighborhoods in subsequent plays. The system is easy to get into an absorbing state: all agents are *D* for large values of *t*, which is known as the tragedy of the *commons* [17].

Recently, Szabó *et al.* [8,11,13] developed the SPDG with voluntary participation, in which agents can take one of three possible strategies, *cooperator, defector* and *loner* (*L*). Cooperators and defectors are interested in taking part in the game and the payoffs for their encounters are assigned as before. *Loners* do not participate in the game temporarily and get the same small fixed income σ ($\sigma<r<t$) as their neighbors. Thus the payoff matrix can be tabulated as

$$
\begin{array}{c|ccc}
 & \mathbf{C} & \mathbf{D} & \mathbf{L} \\
\hline
\mathbf{C} & r & s & \sigma \\
\mathbf{D} & t & p & \sigma \\
\mathbf{L} & \sigma & \sigma & \sigma.
\end{array}
\tag{1}
$$

Each element in the matrix denotes the corresponding payoff of an agent adopting the strategy of the left and encountering an agent performing the strategy of the above. In the volunteers' version, the three strategies can coexist by cyclic dominance (*D* invades *C* invades *L* invades *D*), which efficiently avoids the system getting into a frozen state.

In this Brief Report, we study the SPDG with voluntary participation in the Newman-Watts (NW) network, which is a typical small-world model constructed as follows: starting with a two-dimensional lattice with periodic boundary conditions; each agent locates on the lattice and links with its four nearest neighbors; for every agent, with probability Q, we add a long range link for each of its four links to a random selected agent from the whole system with duplicate links forbidden; then a NW network is realized (see Ref. [18] for details). The structural characteristics of social communities, namely, high clustering and small diameter, can be well described by this small-world graph. A round of play consists of the encounters of all agents with their nearest neighbors. Following Ref. [14], the payoffs earned by the agents are calculated as average and not accumulated from round to round. To start the next round, agents are allowed to inspect the profits collected by their neighbors and adjust their strategies.

We argue that the ingredients for agents changing their states mainly come from two aspects: (i) For the sake of

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pursuing higher profits, agents have a trend to follow the successful agents who get higher payoffs, i.e., "successful" strategies are imitated. We figure that ith agent adopts the strategy of its arbitrary neighbor j with a probability

$$\gamma_{ij} = \frac{g_j}{\sum\limits_{k \in \Omega_i} g_k}, \qquad (2)$$

where g_j denotes the average profit earned by jth agent and Ω_i is the community composed of the nearest neighbors of i and itself; (ii) When one agent and its neighbors are in the same state and get the same average payoffs, there is a spontaneous willingness to make some mutations. We propose that the agents getting into the above case make spontaneous alterations with a probability depending on the elements of the payoff matrix. If the agent under consideration is C, in the next round, the probabilities for its changing to C, D, or L are $r/(r+t+\sigma)$, $t/(r+t+\sigma)$, and $\sigma/(r+t+\sigma)$ respectively; if the agent is D, the probabilities for its changing to C, D, or L are $s/(s+p+\sigma)$, $p/(s+p+\sigma)$, and $\sigma/(s+p+\sigma)$ respectively; and if the agent is L, the probabilities of its changing to C, D, or L are the same value and equal to $1/3$. This spontaneous mutational mechanism not only efficiently avoids the system getting into a frozen state but also sufficiently describes the agents' flexibility.

Our analysis of the model is based on systematic Monte Carlo (MC) simulations performed in different NW networks with the total size of 200×200 populations. The three strategies are assigned randomly to the agents with probability $1/3$ initially. For convenience, following Refs. [10,11,14,15], we set $s=p=0$, $r=1$, $\sigma=0.3$, and $1<t<2$. We define $t-r$ as the relative temptation quantity (shortly RTQ) reflecting the extent of the temptation and cursorily partition the networks into three regions: lattice, small-world and random graphs corresponding to the variational range of Q: (0.0001, 0.001), (0.001, 0.3) and (0.3, 1) respectively. We iterate the rules of the model with parallel updating. The total sampling times are 5000 MC steps. After appropriate relaxation times the system stabilizes in dynamical equilibrium characterized by their densities of ρ_C, ρ_D, ρ_L and average payoffs P_C, P_D, P_L. According to the previous assumption, it is easy to know that P_L is always equal to σ. All the results are averaged over the realizations of ten networks.
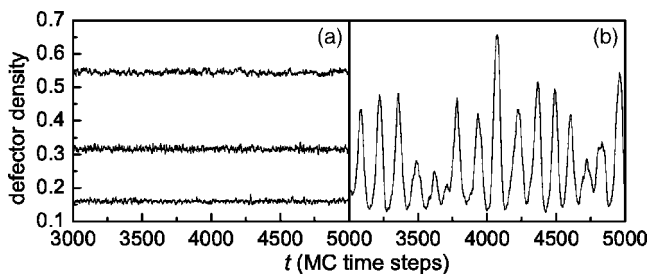
The main features of the steady-state phase diagram can

FIG. 2. MC data of the density of defectors as a function of the network's structure parameter Q under different values of RTQ: 0.02 (a), 0.1 (b), 0.2 (c) and 0.8 (d). Closed squares represent the average density of defectors; open circles and triangles show their maximal and minimal values due to oscillation.

be summarized as follows. All three states coexist and co-evolve steadily in equilibrium state. For large values of Q with very small values of RTQ, strong global oscillations arise, which is similar to the phenomena studied in Ref. [13] for high temptation to defect. The bifurcation of ρ_D for large values of temptation studied in Refs. [11,13], however, does not arise in our model. For small values of Q with arbitrary values of RTQ or large values of RTQ with arbitrary values of Q, the stationary state is characterized by a weak global oscillation where the amplitude of fluctuation is significantly less than the corresponding average value. As a distinct view, in Fig. 1, the last 2000 steps' evolution of ρ_D under values of Q (0.1 and 0.5) and RTQ (0.02 and 0.56) has been tracked (the evolution of ρ_C and ρ_L are similar to ρ_D); the average values of ρ_D and the corresponding maximum and minimum deviation in the steady state are also reported in Fig. 2 for fixed values of RTQ (0.02, 0.1, 0.2, 0.8) with varied values of $Q \in (0.0001 \sim 1.0)$ and in Fig. 3 for fixed values of Q (0.001, 0.1, 0.5, 1.0) with varied values of RTQ $\in$ (0.0–1.0). These phenomena can be explained as follows.

During the process of the evolution, *defectors* cannot form stable large clusters, of which the inner agents would

FIG. 1. Evolution of the density of *defectors* (ρ_D) with varied values of (RTQ, Q) under the equilibrium state: (a) form top to bottom, the curves correspond to (0.02, 0.1), (0.56, 0.1), (0.56, 0.5) respectively; and (b) (0.02, 0.5).
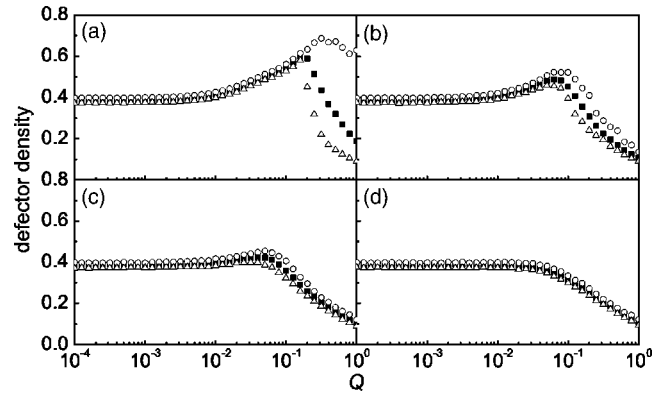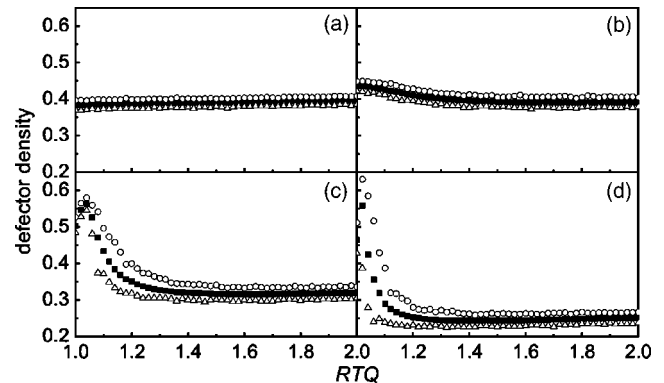
FIG. 3. MC data of the density of defectors as a function of RTQ under different values of the network's structure parameter Q: 0.001 (a), 0.1 (b), 0.5 (c), and 1.0 (d). The symbols as shown in Fig. 2.
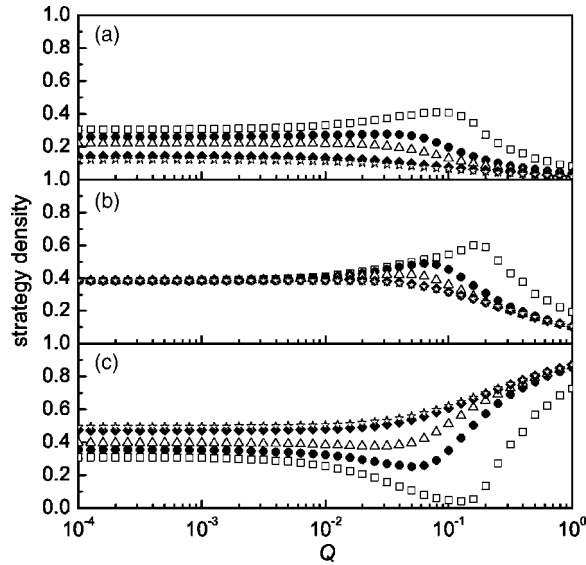
FIG. 4. Densities of *cooperators* (a), *defectors* (b), and *loners* (c) vs the network's structure parameter Q under different values of RTQ. The symbols of open squares, closed circles, open triangles, closed diamonds, and open stars correspond to the value of RTQ: 0.02, 0.1, 0.2, 0.56, and 0.8, respectively.

FIG. 5. Average payoffs of *cooperators* (a) and *defectors* (b) vs the network's structure parameter Q under different values of RTQ: 0.02, 0.1, 0.2, 0.56, and 0.8. The symbols are the same as shown in Fig. 4, and the dotted line indicates the fixed average payoff of *loners*.

get zero profit and possess the same state as their neighbors (*local commons*). According to the evolutionary rules, they will try to throw off embarrassment by changing their strategies. Namely, the easy formation of clusters of D will make the agents self-adapt frequently in their communities, and then confine the fluctuation of ρ_D in a narrow range [see Fig. 1(a), Fig. 2(b), Fig. 2(c), Fig. 2(d), Fig. 3(a), Fig. 3(b)]. There are two factors favoring the forming of clusters of *defectors*: the high temptation to defect (large values of RTQ) and the well clustered structure of the agents (small values of Q), which would strengthen the adoption and the imitation of strategy D greatly. Therefore, in our model, high temptation to defect will only give rise to steady oscillation of the system rather than result in the bifurcation phenomena studied in Refs. [11,13]. While for poorly clustered agents (large values of Q) with low temptation, the formation of large clusters of *defectors* is reasonably difficult, which would slow down the evolutionary velocity of the whole system and guarantee the growth (decline) of ρ_D lasting for a long time, and consequently broaden the fluctuant amplitude [see Fig. 1(b), Fig. 2(a), Fig. 3(c) and Fig. 3(d)].

In addition, in the lattice region, ρ_D keeps a steady level for any values of RTQ [see Fig. 2, Fig. 3(a), and Fig. 4(b)]. It is also a result of the fast self-adaptation of the agents. With the increasing of RTQ, agents of C are easy to change to D for high temptation, and then again change to L because clusters of *defectors* are extremely unstable and cannot survive a long time. The decrease of ρ_C nearly results in the increasing of ρ_L [see Fig. 4(a) and Fig. 4(c)]. In this region, the fast self-adaptation of the agents also leads to the case that the neighbors of *defectors* would include other types of agents in most time during the evolution, which gives rise to larger values of P_D than P_L. By comparison, in Refs. [11,13], very big clusters of *defectors* can survive a long time during the evolution and most agents would get only the zero payoff
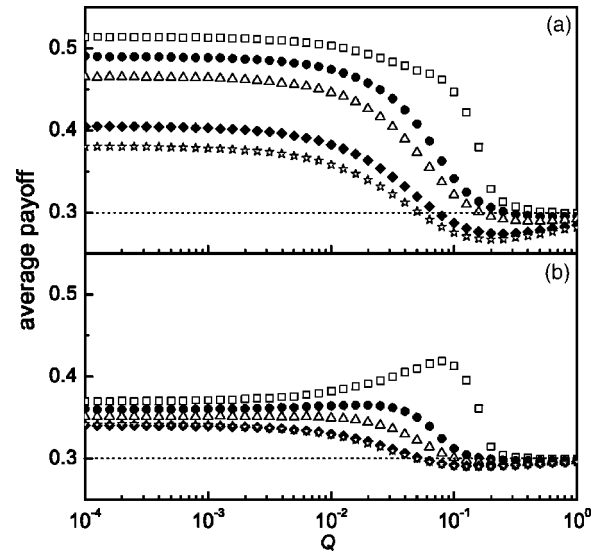
resulting in lower average payoffs of the *defectors* than the *loners*. It is obvious that the differences in the evolutionary dynamics of the game give rise to the distinct results. It is worth mentioning that the present model is also different from the cyclic spatial games studied in Ref. [19] where the dynamics evolution is governed by a strictly cyclic dominance, i.e., *rock* dominates *scissors* dominates *paper* dominates *rock*. While in our model, any two types of the three strategies can transform each other in a particular case. As a result of the difference in evolutionary dynamics, the phase transitions phenomena studied in Ref. [19] for *rock-scissors-paper* games do not arise in our model.

Another interesting feature of the equilibrium phase diagram is that in the vicinity of $Q=0.1$ where the NW networks possess notable small-world effect, namely, large clustering and small diameter at the same time, agents are willing to participate in the game in the case of very low temptation to defect. To view in detail, in Fig. 4 and Fig. 5, we plot the average densities and corresponding average payoffs of *C-D-L* vs the small-world parameter Q under different values of RTQ, respectively. For very low temptation to defect (e.g. RTQ=0.02), the evolutionary curve of ρ_L decreases slowly with the increasing of Q and reaches a minimum at a certain culminating point. As Q increases over this point, ρ_L ascends rapidly (see Fig. 4). We conclude that two factors, the very low temptation to defect and the small-world property of the network, are beneficial for the spreading of C in the system, which then stimulates more and more agents to take part in the game. In the case of more random networks ($Q \rightarrow 1$), the evolutionary results of the game are qualitatively the same as Refs. [11,13], i.e., the majority of members in the system are *loners* and the values of P_C and P_D get closed to the fixed value σ.

In summary, we have studied the SPDG with voluntary participation in NW small-world networks. To model the re-

alistic social systems, some reasonable ingredients are introduced to the evolutionary dynamics: each agent in the networks is a pure strategist and can only take one of three strategies (C,D,L); its strategical transformation is associated with both the number of strategical states and the magnitude of average profits, which are adopted and acquired by its coplayers in the previous round of play. To model initiative and flexibility, a stochastic strategy change is applied when the agents get into the condition of *local commons*. The agents self-adapt and self-organize into dynamical equilibrium after a short transient. When the agents are well structured (the cases of small values of Q), they can steadily coexist and coevolve. On the other hand, for high temptation or more random networks, *loners* dominate the network. Especially, in the case of very low temptation to defect, it is found that agents are willing to participate in the game in typical small-world regions and intensive collective oscillations arise in more random regions.

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